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The Supergravity Dual of the BMN Matrix Model

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Abstract

We propose type IIA supergravity solutions dual to the 1/2 BPS vacua of the BMN matrix model. These dual solutions are analyzed using the Polchinski-Strassler method and have brane configurations of concentric shells of D2 branes (or NS5 branes) with various radii and D0 charge. These branes can be viewed as polarized from N D0 branes by a transverse R-R magnetic 6-form flux and an NS-NS 3-form flux. In the region far from branes, the solutions reduce to perturbation around the near horizon geometry of N D0 branes, by turning on these R-R and NS-NS fluxes, which are dual to the deformation of the BFSS matrix model by adding mass terms and the Myers term. The solutions with these additional fluxes preserve 16 supersymmetries. We also briefly discuss these fluxes in the possible supergravity duals of M(atrix) theories on less supersymmetric plane-waves.

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1 Introduction

The AdS/CFT correspondence [1] provides a remarkable method to study the physics of gauge theories by their dual descriptions in string theory and its low energy limit–supergravity. In this paper, we will utilize this correspondence to study the dual descriptions of the BMN matrix model [2] and its 1/2 BPS classical vacua by IIA supergravity solutions. This model can be considered as a 0+1 dimensional $U(N)$ gauge theory which has a discrete vacuum spectrum and can serve as a relatively simple example of a theory with a finite number of vacua at finite N. This model has very similar vacuum structure to that of the $\mathcal{N} = 1^*$ theory [3], whose string theory dual was constructed by the Polchinski-Strassler solution [4].

The BMN matrix model [2] was conjectured to be the DLCQ of M theory on the 11-d maximally supersymmetric plane-wave background [5], [6]. The action of this model can be obtained[1]either by matrix-regularization of a supermembrane action (e.g. [7]) on the 11-d plane-wave background, or from the quantum mechanics of N D0 branes on the background of the 11-d plane-wave compactified to 10 dimensions [8]. The classical solutions and quantum spectrum of this model have been extensively studied by e.g. [8], [9], [10]. This model may also be thought of as a deformation of the BFSS matrix model [11] by adding mass terms and Myers term to the Lagrangian. Due to these terms, the plane-wave background removes the flat directions of the BFSS matrix model and the wave-functions of the D0 branes no longer spread uniformly over the space but are instead localized around some fuzzy spheres. The 1/2 BPS classical supersymmetric solution describes D0 branes sitting at the origin of a 6-dimensional subspace as a result of mass terms in these directions and in another 3-dimensional subspace, their matrix-coordinates obey the $SU(2)$ commutation relations as a result of the interplay between mass terms and the Myers term [2], [8]:

$$[X^i, X^j] = i\frac{\mu}{3}\epsilon_{ijk}X^k, \qquad i, j, k = 1, 2, 3, \tag{1}$$

where μ is a mass parameter. The coordinates X^i $(i = 1, 2, 3)$ are $N \times N$ matrices and therefore their solutions are in N-dimensional representations of the Lie group $SU(2)$. Since for each positive integer n there is an irreducible n-dimensional representation of $SU(2)$, each vacuum solution can be labeled by a partition of the integer N into positive

[1]The action of the BMN matrix model can also be obtained by the dimensional reduction of $d = 4$, $\mathcal{N} = 4$ $U(N)$ SYM on $R \times S^3$ keeping certain $SU(2)$ invariant Kaluza-Klein modes on S^3 [14].

integers n_i, with $\sum_i n_i = N$, corresponding to the direct product of these n_i-dimensional irreducible representations. So the D0 branes form a collection of fuzzy spheres with radii proportional to n_i. For large n_i, these fuzzy spheres can be well-approximated as round spheres up to a non-commutativity correction [12].

These vacua have similar structure to those of the $\mathcal{N} = 1^*$ theory [3], which is a deformation of $d = 3 + 1$, $\mathcal{N} = 4$ $U(N)$ SYM by adding mass terms for the 3 chiral multiplets Φ_i $(i = 1, 2, 3)$ to the superpotential and the $\mathcal{N} = 4$ supersymmetry is broken to $\mathcal{N} = 1$. As a result, the F-term equations for the classical supersymmetric vacua yield[2] [3]:

$$[\Phi_i, \Phi_j] = -i\frac{m}{\sqrt{2}}\epsilon_{ijk}\Phi_k, \qquad i, j, k = 1, 2, 3. \tag{2}$$

Since Φ_i are $N \times N$ traceless matrices, the solutions are also in N-dimensional representations of $SU(2)$ and each vacuum is also labeled by a partition of the integer N into positive integers n_i with $\sum_i n_i = N$, in the same way as in the BMN matrix model.

Due to their similarities, many aspects of the BMN matrix model and the $\mathcal{N} = 1^*$ theory may be studied in parallel both in the field theory context and in their dual supergravity descriptions. They can both be studied from the point of view of deformation by relevant operators around an originally undeformed theory in the U.V. region. They both have fuzzy sphere vacua which can be interpreted in dual supergravity solutions as branes of higher dimensionality polarized from branes of lower dimensionality.

The various Polchinski-Strassler solutions [4] in the string dual of the $\mathcal{N} = 1^*$ theory have brane configurations corresponding to D3 branes polarized into various D5 branes (in the weak coupling regime) or NS5 branes (in the strong coupling regime) via Myers' dielectric effect [15]. On the near horizon geometry of N D3 branes, i.e. $AdS_5 \times S^5$, Polchinski and Strassler [4] found that one can turn on additional R-R 3-form fluxes and NS-NS 3-form fluxes to polarize the D3 branes into D5 or NS5 branes with world-volumes $R^4 \times S^2$. They found that the radii of the S^2 of these D5 branes are proportional to the D3-charge that these D5 branes carry, under certain approximations.

In this paper, we find similar physics happened in terms of D0 branes. We start from the near horizon geometry of N D0 branes which is dual to the BFSS matrix model [16], and find that one can turn on additional transverse magnetic R-R 6-form flux and NS-NS 3-form flux whose Hodge duals can couple to D2 or NS5 charge and thereby to cause the N D0 branes to polarize into various D2 or NS5 branes. We find that the D2 branes

[2]The three Φ_i are rescaled to make the three masses equal.

polarized from the D0 branes in the presence of these additional fluxes have S^2-wrapped world-volumes and the radii of their S^2 are proportional to the D0-charge they carry, under certain approximations. We find that the general equilibrium brane configuration could consist of many concentric D2 branes each with its radius proportional to its D0-charge.

We thus propose a holographic dual description of the 1/2 BPS classical vacua of the BMN matrix model, using dual IIA solutions with brane configurations, in the cases when all n_i are large. In the appropriate regimes of parameters, there is a one-to-one correspondence between the supergravity solutions in the bulk, where there are concentric branes carrying D0-charge, and the 1/2 BPS classical vacua of the BMN matrix model on the boundary, which are collections of fuzzy spheres. The concentric branes are either D2 branes or NS5 branes, in the regimes of weak or strong effective 't Hooft couplings, respectively, in the matrix perturbation theory of the BMN matrix model [8]. In the D2 brane descriptions, each dual supergravity solution corresponds to a way of dividing up and distributing the total D0-charge N to several D2 branes each with D0-charge n_i, by an identical partition of N in the matrix model side, in terms of fuzzy spheres. On the other hand, in the NS5 brane descriptions, each dual supergravity solution also corresponds to a way of dividing up and distributing the total D0-charge N to several NS5 branes, but by a dual partition[3] of N in the matrix model side. These concentric branes in the supergravity side are holographically mapped to the fuzzy spheres in the matrix model side. In the large r region of the supergravity solutions (where r is the radial variable of the 9 spatial dimensions), the additional R-R 6-form flux and NS-NS 3-form flux are dual to the deformation of the BFSS matrix model by adding mass terms and the Myers term.

These solutions of IIA when lifted up to 11 dimensions describe supergravitons polarized into M2 or M5 branes. They are giant gravitons (e.g. [17], [18], [19]) each carrying a fraction of the total light-cone momentum. The light-cone momentum of the N supergravitons $p_+ = \frac{N}{R}$ are shared to several M2 branes each with light-cone momentum $p_+^{(i)} = \frac{n_i}{R}$ and with radius proportional to n_i, in the same way as a partition of integer N. In the M5 brane description, it is also a way of sharing the total light-cone momentum to several M5 branes but by a dual partition of integer N [13].

The main body of the paper will focus on the details of construction of the dual supergravity solutions in terms of D2 brane configurations by the Polchinski-Strassler method, valid in the regimes of weak effective 't Hooft couplings in the matrix perturbation

[3] Here, a dual partition of N is defined via switching the rows and columns of a Young tableau, see e.g. [13] p.6.

theory of the BMN matrix model [8]. In the next section, we study the R-R 6-form flux G_6 and NS-NS 3-form flux H_3, in the large r region, as perturbation around the near horizon geometry of N D0 branes. In section 3, we study the equilibrium radii of the S^2 -wrapped D2 branes with D0-charge in the solutions with general brane configurations. In section 4, we solve the fluxes H_3 and G_6 as being sourced by these polarized D0 branes with D2-charge. In section 5, we study the metric and dilaton near each shell of the branes as well as in the large r region. In the last section, we discuss related issues to our results and also possible generalizations to the supergravity duals of M(atrix) theories on less-supersymmetric plane-waves.

2 The R-R 6-form flux and NS-NS 3-form flux in the large r region

Since the BMN matrix model can be considered as a deformation of the BFSS matrix model, its dual supergravity solutions, in the large r region, can be considered as perturbation around the near horizon geometry of N D0 branes, which in string frame is (e.g. [22], [23]):

$$\begin{aligned} ds^2 &= -Z^{-1/2}dt^2 + Z^{1/2}d\overrightarrow{x}_i^2, \qquad i = 1, ..., 9, \\ e^{\Phi} &= g_s Z^{3/4}, \qquad C_1 = g_s^{-1}(Z^{-1} - 1)dt, \qquad Z = \frac{R^7}{r^7}, \end{aligned} \tag{3}$$

where $R^7 = 60\pi^3 g_s N \alpha'^{7/2}$.

The fluctuations around this background we are interested in are the R-R flux $\widetilde{F}_4$ and NS-NS flux H_3 which are relevant to the couplings to the D2 or NS5 branes that can be polarized from D0 branes. When these additional fluxes in large r region can be considered as small fluctuations around the above background (3), it's easy to see that the perturbations of the metric, dilaton and F_2 are all of second order or higher in the fluctuations. Therefore if we neglect quantities that are of second order or higher in the fluctuations, we only need to turn on these additional fluxes without giving corrections to the background. For convenience, we can dualize the 4-form flux $\widetilde{F}_4$ into a transverse[4] 6-form flux G_6 via $G_6 = Z^{3/8} * \widetilde{F}_4$, where $*$ is the Hodge dual with respect to the 10d

[4]By transverse we mean that the forms such as H_3 and G_6 etc. have all components transverse to the D0 brane world-volume.

metric in Einstein frame. After some derivation in appendix A, the equations of motion for the transverse fluxes H_3 and G_6 turned out to possess a simple form:

$$dH_3 = 0, \quad (4)$$

$$dG_6 = 0, \quad (5)$$

$$d[Z^{-1}(H_3 - g_s *_9 G_6)] = 0, \quad (6)$$

$$d[Z^{-1}(*_9 H_3 - g_s G_6)] = 0, \quad (7)$$

where $*_9$ is the Hodge dual in the transverse 9-d with respect to a flat 9-d metric. These constraints tell us that H_3 and G_6 are both closed forms and $Z^{-1}(H_3 - g_s *_9 G_6)$ is annihilated by both d and $d*_9$ in the transverse 9 dimensions.

The solution should break the isometry $SO(9)$ to $SO(3) \times SO(6)$, where the $SO(3)$ is the isometry of the 123 subspace and the $SO(6)$ is the isometry of the other 6-d transverse subspace. According to this isometry, in the large r, we should look for the fluxes of the form

$$H_3 = r^m(\alpha T_3 + \beta V_3), \quad (8)$$

$$G_6 = r^n(\gamma *_9 T_3 + \delta *_9 V_3), \quad (9)$$

where $T_3 = dx^1 \wedge dx^2 \wedge dx^3$ is the volume form of the 123 subspace and V_3 is defined as $V_3 = d\ln r \wedge S_2$, where $S_2 = \frac{1}{2!}\varepsilon_{ijk}x^i \wedge dx^j \wedge dx^k$, $(i, j, k = 1, 2, 3)$ and $m, n, \alpha, \beta, \gamma, \delta$ are constants.

By plugging this ansatz, the set of equations (4)-(7) admit four solutions in two pairs (see appendix B). Each pair consists of one non-normalizable and one normalizable solution as follows[5]:

first pair :

$$H_3 = \alpha r^{-7}(T_3 - \frac{7}{3}V_3), \qquad G_6 = g_s^{-1}\alpha r^{-7}(\frac{1}{3} *_9 T_3 - \frac{7}{3} *_9 V_3), \quad (10)$$

$$H_3 = \alpha r^{-9}(T_3 - 3V_3), \qquad G_6 = g_s^{-1}\alpha r^{-9}(*_9 T_3 - 3 *_9 V_3), \quad (11)$$

second pair :

$$H_3 = \alpha T_3, \qquad G_6 = g_s^{-1}\alpha *_9 T_3, \quad (12)$$

$$H_3 = \alpha r^{-16}(T_3 - \frac{16}{3}V_3), \quad G_6 = g_s^{-1}\alpha r^{-16}(-\frac{5}{3} *_9 T_3 + \frac{8}{3} *_9 V_3), \quad (13)$$

[5]The solution with the $n = 0$ fluxes in (12), when uplifted to 11d, leads to the solution of the "superposition" of the 11d gravitational wave and 11d plane-wave, as described in [6] p.20, [25].

where the α in different lines are different. In the language of AdS/CFT correspondence, the pair of $n=-7$ and $n=-9$ solutions[6], i.e. (10), (11), corresponds to the operators of mass deformation in the matrix model side and the VEV of it respectively [24]. As will be shown in section 4, in the large r region, H_3 and G_6 are the superpositions of the $n=-7$ and $n=-9$ solutions, while the $n=-7$ solution is of first order in the mass parameter μ of the BMN matrix model, and the $n=-9$ solution is of third order in the mass parameter μ (see appendix F).

Since our solutions are dual to 1/2 BPS classical vacua of BMN matrix model, we should have 16 supersymmetries in our solutions. This is one of the differences between the dual solutions of fuzzy sphere vacua of BMN matrix model and those of the $\mathcal{N}=1^*$ theory. In the Polchinski-Strassler case, the supersymmetry is broken from $\mathcal{N}=4$ to $\mathcal{N}=1$ [26], [27], while in our case the solutions after turning on H_3 and G_6 still preserve 16 supersymmetries.

We therefore explicitly solved the Killing spinor perturbatively in first order in μ, when we turn on the fluctuations of the $n=-7$ solution (10) of H_3 and G_6, which are of first order in μ. The Killing spinor before perturbation is the Killing spinor $\epsilon^{(0)}$ in the near horizon geometry of N D0 branes, and the Killing spinor, after turning on H_3 and G_6 that are of first order in μ, could be written perturbatively as $\epsilon=\epsilon^{(0)}+\epsilon^{(1)}$, where $\epsilon^{(1)}$ is the perturbation of the Killing spinor, and is of first order in μ.

We can thereby split the gravitino equations and the dilatino equations order by order in μ, and the equations for the first two orders in μ are[7] (in string frame [28], see appendix C):

$$\begin{aligned}\left(\frac{1}{2}\Gamma^m\partial_m\Phi+\frac{3}{8}e^{\Phi}\ \not{F}_2\Gamma^{\underline{11}}\right)\epsilon^{(0)} &= 0, & (14)\\ \left(\partial_m+\frac{1}{4}\omega_{mab}\Gamma^{ab}+\frac{1}{8}e^{\Phi}\ \not{F}_2\Gamma_m\Gamma^{\underline{11}}\right)\epsilon^{(0)} &= 0, & (15)\end{aligned}$$

$$\begin{aligned}\left(\frac{1}{2}\Gamma^m\partial_m\Phi+\frac{3}{8}e^{\Phi}\ \not{F}_2\Gamma^{\underline{11}}\right)\epsilon^{(1)} &= -\left(\frac{1}{4}\not{H}_3\Gamma^{\underline{11}}+\frac{1}{8}e^{\Phi}\widetilde{\not{F}}_4\right)\epsilon^{(0)}, & (16)\\ \left(\partial_m+\frac{1}{4}\omega_{mab}\Gamma^{ab}+\frac{1}{8}e^{\Phi}\ \not{F}_2\Gamma_m\Gamma^{\underline{11}}\right)\epsilon^{(1)} &= -\left(\frac{1}{8}H_{mab}\Gamma^{ab}\Gamma^{\underline{11}}+\frac{1}{8}e^{\Phi}\widetilde{\not{F}}_4\Gamma_m\right)\epsilon^{(0)}. & (17)\end{aligned}$$

[6] Here n is the scaling dependence of the fluxes on r in large r region.

[7] The letters with a slash denote the contractions of forms with gamma matrices, for example: $\not{F}_2=\frac{1}{2!}F_{ab}\Gamma^{ab}$, $\not{H}_3=\frac{1}{3!}H_{abc}\Gamma^{abc}$, $\widetilde{\not{F}}_4=\frac{1}{4!}\widetilde{F}_{abcd}\Gamma^{abcd}$.

The first pair of equations (14), (15), i.e. the zeroth order equations, give the unperturbed $\epsilon^{(0)}$ which is known in the literature (e.g. [29], [30]):

$$\epsilon^{(0)} = Z^{-1/8}\eta, \tag{18}$$

where η is a constant spinor with definite helicity via projection condition $(1+\Gamma^{\underline{0}}\Gamma^{\underline{11}})\eta = \eta$. Input this result into the other pair of equations (16), (17), i.e. the first order equations, we find that the first order correction $\epsilon^{(1)}$ can be separated into a time-dependent part $\epsilon_1^{(1)}$ and a time-independent part $\epsilon_2^{(1)}$, and the former has the same helicity with the unperturbed $\epsilon^{(0)}$ while the latter has the opposite helicity to $\epsilon^{(0)}$.

Explicit calculations are in appendix C and the results are[8]

$$\epsilon = \epsilon^{(0)} + \epsilon_1^{(1)} + \epsilon_2^{(1)}, \tag{19}$$

$$\epsilon_1^{(1)} = \frac{1}{12}\cdot\frac{1}{3!}Z^{-1}[H_{lmn} - g_s(*_9 G_6)_{lmn}]\Gamma^{\underline{lmn}}\epsilon^{(0)}t = -\frac{1}{12}\widetilde{\mu}(\Gamma^{\underline{123}}t)\epsilon^{(0)}, \tag{20}$$

$$\epsilon_2^{(1)} = \widetilde{\mu}(\frac{1}{6}\Gamma^{\underline{i}}x^i - \frac{1}{12}\Gamma^{\underline{a}}x^a)Z^{1/2}\Gamma^{\underline{123}}\Gamma^{\underline{0}}\epsilon^{(0)}, \tag{21}$$

where the fluxes in first order in μ are

$$H_3 = \frac{3}{2}\widetilde{\mu}Z(-T_3 + \frac{7}{3}V_3), \qquad G_6 = \frac{3}{2}g_s^{-1}\widetilde{\mu}Z(-\frac{1}{3} *_9 T_3 + \frac{7}{3} *_9 V_3), \tag{22}$$

and $Z = \frac{R^7}{r^7}$.

We see from (22) that the leading terms of the fluxes H_3 and G_6 in the large r region are proportional to the total D0-charge N of the branes and are independent of the specific brane configurations in the small r region. Note that the equation for the time-dependent part $\epsilon_1^{(1)}$ yields a constraint that the components $Z^{-1}[H_{lmn} - g_s(*_9 G_6)_{lmn}]$ should be constants and therefore $Z^{-1}(H_3 - g_s *_9 G_6)$ need to be a constant 3-form in the transverse 9-d (appendix C). This is indeed the case, since it equals to $-\widetilde{\mu}T_3$.

So we have checked that our solutions with the fluxes in large r region in first order in μ, i.e. the $n = -7$ solution, are supersymmetric, preserving the 16 supersymmetries. In

[8] In the expressions of the Killing spinors (20), (21), the various indices are: The indices l, m, n denote 1,...,9; the indices i denotes 1,2,3 and the indices a denotes 4,...,9; the indices of gamma matrices with a bar below are the gamma matrices in tangent space. Here we consistently use another parameter $\widetilde{\mu}$ in all the expressions instead of the mass parameter μ in BMN matrix model. They are proportional to each other, i.e. $\widetilde{\mu} = \zeta\mu$, where ζ is a dimensionless factor of order 1, which may be figured out by comparing the D2 potential (in section 3) with the matrix model action.

section 4, we will see that the $n = -9$ solution is of third order in μ in large r region. In order to check the supersymmetry when superposing the $n = -7$ and $n = -9$ in large r region, we need to give second and third order corrections to the metric, dialton and F_2. One difference from the original Polchinski-Strassler solutions [4] is that we still preserve all the supersymmetries of the unperturbed background after adding on the additional fluxes. This is not surprising since the T_3 in the expressions of our fluxes is maximally symmetric under the isometry $SO(3) \times SO(6)$. The expressions for the fluxes H_3 and G_6, as well as the Killing spinor in this section, are valid in large r region where the additional fluxes can be considered as small fluctuations compared to the background. The H_3 and G_6 near the brane sources will be discussed in section 4.

3 The position of D2 branes with D0 charge in the small r region

In last section, we have studied the fluxes in the large r region, which are dual to the operators of deformation in matrix model side. In this section, we will consider the small r region, where there are branes which are the holographic maps of the fuzzy spheres in the matrix model side. The general brane configurations in our solutions are concentric shells of p_i D2 branes each with q_i D0-charge, where i denotes the ith shell, and $\sum_i p_i q_i = N$. We will show that the radii of these D2 branes in our solution are proportional to the D0-charge q_i that they carry, under certain approximation.

In a general brane configuration corresponding to the partition $N = \sum_i p_i q_i$, each brane is in an equilibrium position under the potential it feels in the presence of all branes. The equilibrium radius of each brane corresponds to the location of the minimum of the potential that the brane feels. Before we calculate the potential that each brane feels in a general brane configuration, we will first solve a simpler problem. We will calculate the potential of a probe D2 brane with D0-charge q in the background of the near horizon geometry of single-center N D0 branes, with the $n = -7$ additional fluxes H_3 and G_6, i.e. (22), turned on. Then we can generalize the result for the probe brane to the branes that are not probes, in a general brane configuration.

The reason we can make this generalization is that there is certain configuration-independence in our solutions, for example, as showed in section 2, the 3-form $Z^{-1}(H_3 - g_s *_9 G_6)$ remains a constant form at large r, independent of what the brane configurations

are in the small r region. In this section, we will show, under certain approximations, i.e. condition (30), the brane potential is also configuration-independent. It only depends on the radius of the S^2 of the brane and the D0-charge of it, and doesn't care about how the other branes distribute, under this approximation.

In the probe calculation, we require that the D0-charge q it carries is much smaller than the background D0-charge N so that it can be treated as a probe. However, we can relax this condition later when we make the generalization, due to the configuration-independence. The brane will take the shape of an S^2 embedded in 123 subspace and at the origin of the other six transverse dimensions. The DBI and WZ action of the D2 brane with q units of D0-charge in the string frame is

$$S_{D2} = -\tau_{D2} \int d^3\sigma e^{-\Phi} \sqrt{-\det(G_{\alpha\beta} + 2\pi\alpha' \mathcal{F}_{\alpha\beta})} + \tau_{D2} \int (C_3 + 2\pi\alpha' \mathcal{F}_2 \wedge C_1), \tag{23}$$

where $2\pi\alpha' \mathcal{F}_2 = 2\pi\alpha' F_2 - B_2$. We choose the static gauge that the world-volume coordinates are the same as the space-time ones, i.e. t, θ, φ, where the angles parameterize the S^2. The radius of the S^2 in 123 subspace is denoted as r_1. The D0-charge q of the D2 brane is realized as a world-volume 2-form flux $F_2 = \frac{1}{2} q \sin\theta d\theta \wedge d\varphi$. The B_2 is given by the $n = -7$ solution (22) of the H_3 in section 2.

We will make an approximation that is similar to Polchinski-Strassler [4] that the dominating terms in both DBI and WZ actions are from the contribution of F_2, which requires the conditions (see appendix D):

$$\frac{4\pi^2\alpha'^2 \det F_2}{\det G_\perp} = \frac{\pi^2\alpha'^2 q^2 r_1^3}{R^7} \sim \frac{q^2 r_1^3}{g_s N} \gg 1, \qquad \frac{2\pi\alpha' F_{\alpha\beta}}{B_{\alpha\beta}} = -\frac{2\pi\alpha' q r_1^4}{\widetilde{\mu} R^7} \sim \frac{q r_1^4}{g_s N} \gg 1, \tag{24}$$

where $G_\perp$ is the pull-back metric parallel to the S^2. Under this approximation we can expand the square-root in the DBI action around $\det F_2$ and then the leading terms in the DBI and WZ actions precisely cancel (appendix D):

$$-\tau_{D2} g_s^{-1} \int d^3\sigma Z^{-3/4} \sqrt{-\det G_{\shortparallel}} \cdot 2\pi\alpha' \sqrt{\det F_2} + \tau_{D2} \int 2\pi\alpha' F_2 \wedge C_1 = 0, \tag{25}$$

where we choose the gauge $C_1 = g_s^{-1} Z^{-1} dx^0$ and $G_{\shortparallel}$ is the pull-back metric parallel to time. This is because that the leading terms in the DBI and WZ actions, which are both contributed from the F_2, describe the potential between D0 and D0 charges and it should be zero due to supersymmetry. The two leading terms are both large but they cancel.

The subleading terms of the DBI and WZ actions are respectively:

$$-\tau_{D2} g_s^{-1} \int d^3\sigma Z^{-3/4} \frac{\sqrt{-\det G_{\shortparallel}} \det G_\perp}{4\pi\alpha' \sqrt{\det F_2}} = \int dt \left(-\frac{2\tau_{D2} r_1^4}{g_s \alpha' q} \right), \tag{26}$$

$$\tau_{D2} \int C_3 = \int dt \left(\frac{4\pi \tau_{D2} \widetilde{\mu} r_1^3}{3 g_s} \right), \tag{27}$$

where $C_3 = \frac{1}{3} g_s^{-1} \widetilde{\mu} dx^0 \wedge S_2$. These two terms in the brackets are of order r_1^4 and $\widetilde{\mu} r_1^3$ respectively, which can be identified with the commutator term $\mathrm{Tr}\left(\frac{1}{4}[X^i, X^j]^2\right)$ and the Myers term $-\mathrm{Tr}\left(i\frac{\mu}{3}\epsilon_{ijk} X^i X^j X^k\right)$ in the matrix model Lagrangian. There should be another term of order $\widetilde{\mu}^2 r_1^2$ coming from the second order corrections of the metric, dilation and C_1, which can be identified as $-\mathrm{Tr}\,\frac{1}{2}\left(\frac{\mu}{3}\right)^2 (X^i)^2$ in the matrix model Lagrangian. The subleading terms of the DBI and WZ actions, plus the second order corrections from the metric, dilation and C_1 are expected to complete the potential with a perfect square, due to supersymmetry condition. This is not surprising since this corresponds to the perfect-square term $-\int dt\, \frac{1}{2}\mathrm{Tr}\left(\frac{\mu}{3}X^i + i\epsilon^{ijk} X^j X^k\right)^2$ in the matrix model action [8]. So the third term could be in principle read off from the first two terms.

By the approximation (24) and supersymmetry condition, the action should then be

$$S \approx -\int dt \frac{2\tau_{D2}}{g_s \alpha' q} \left(r_1^2 - \frac{\pi}{3} \widetilde{\mu} \alpha' q r_1 \right)^2. \tag{28}$$

So we see that the brane potential depends on the radius of the S^2, i.e. r_1, and its D0-charge q, and is independent of the warp-factor Z under the approximation (24). The subleading term in DBI action is Z-independent since both Z factors from numerator and denominator exactly cancel (appendix D). The subleading term in WZ action is also Z-independent because C_3 is Z-independent. And the third term should also be Z-independent due to supersymmetry. Therefore the potential of the brane only cares about its D0-charge and is independent of the brane configuration of the solution under this approximation. The potential of a non-probe D2 brane with D0-charge in a general brane configuration is thus still of the form (28). There is a non-trivial equilibrium radius at

$$r_0 \approx \frac{\pi}{3} \widetilde{\mu} \alpha' q, \tag{29}$$

which is proportional to the D0-charge q. This is similar to the matrix model description of the fuzzy spheres that each of them has a radius proportional to the size q of its block matrix-coordinates [2], [8], up to a non-commutativity correction [12].

After knowing the equilibrium radius r_0, we get the consistent condition for our approximation from (24) that

$$\frac{q^5}{g_s N} \gg 1. \tag{30}$$

This is a kind of scaling bound between q and $g_s N$, where q is the D0-charge of the D2 brane and N is the background D0-charge[9]. The approximation (30) actually guarantees that the contributions to the DBI and WZ actions are both from the D0-charge but they cancel and then the remaining terms form a perfect square and is independent of the brane configuration and the warp-factor.

So in our general brane configurations, suppose there are concentric shells of p_i D2 branes each with D0-charge q_i (the label i denotes the ith shell), the total potential of all the branes can be considered as the sum of individual potentials and we have:

$$S \approx -\int dt \sum_i \frac{2\tau_{D2} p_i}{g_s \alpha' q_i} \left(\left(r_1^{(i)} \right)^2 - \frac{\pi}{3} \widetilde{\mu} \alpha' q_i r_1^{(i)} \right)^2 , \tag{31}$$

where $r_1^{(i)}$ denote the radii of the S^2 of the branes on the ith shell and their equilibrium radii $r_0^{(i)}$ are therefore

$$r_0^{(i)} \approx \frac{\pi}{3} \widetilde{\mu} \alpha' q_i, \qquad \qquad \sum_i p_i q_i = N. \tag{32}$$

Note that there could be coincident p_i D2 branes on the same shell if they have the same amount of D0-charge q_i, which corresponds to some copies of the irreducible representations of the $SU(2)$ of the same dimension, in the matrix model side.

4 The additional fluxes in the presence of polarized sources

In section 2, we focused on the large r region and found out the additional fluxes H_3 and G_6, as perturbations around the near horizon geometry of N D0 branes. In section 3, we have studied the situations in the small r region and figured out the radius of each D2 brane, in general configurations. The leading terms of these fluxes H_3 and G_6 in the large r region depend on the total D0-charge of all the branes and are independent of the brane configurations, while in the small r region these fluxes are dependent on specific brane configurations. The expressions for the fluxes in section 2 are not valid near the brane

[9] This is very similar to the condition for the approximation in the original Polchinski-Strassler solution, i.e. $\frac{n^2}{g_s N} \gg 1$ [4], where n was the D3-charge of the D5 brane and N was the background D3-charge. We have different powers of n (or q) because the powers of the r in the warp-factor Z is different. In their case $Z = \frac{R^4}{r^4}$, while in our case $Z = \frac{R^7}{r^7}$. The warp-factor dilutes the background charge so the power dependence of n (or q) for different Z is different.

sources. In this section we will therefore study the behavior of H_3 and G_6 in the presence of these sources.

For simplicity of the calculation, we study the special case when there is only a single shell of D2 branes with total D0-charge N. Suppose we have p coincident D2 branes each with D0-charge $q = N/p$, so the radius of the shell is $r_0 \approx \frac{\pi}{3}\widetilde{\mu}\alpha'(N/p)$. We study the case that the total D2-charge p is small, so the background metric can be approximated by the near horizon geometry of multi-center D0 branes distributed on the S^2 with radius r_0.

The equation for $*\widetilde{F}_4$ will have a source term due to the D2-charge. We are interested in the H_3 and G_6 on the background of the near horizon geometry of a shell of multi-center D0 branes with small D2-charge turned on. We are now doing perturbation in terms of small parameter p. The warp-factor $Z = \frac{R^7}{r^7}$ is replaced by the multi-center warp factor Z_1 in solution (3):

$$Z_1 = \frac{R^7}{10 r_1 r_0}\left[\frac{1}{[(r_1 - r_0)^2 + r_2^2]^{5/2}} - \frac{1}{[(r_1 + r_0)^2 + r_2^2]^{5/2}}\right], \tag{33}$$

which is the superposition of harmonic functions, corresponding to D0-charge uniformed distributed on an S^2 with radius r_0 in the 123 subspace and centered at the origin of the other 6-d transverse subspace. Here r_1 is the radius of 123 subspace and r_2 is the radius of the other 6-d transverse subspace. Its easy to see that Z_1 reduces to $Z = \frac{R^7}{r^7}$ at large r.

The IIA SUGRA equations for H_3 and G_6 on the background of the near horizon geometry of a single shell of multi-center D0 branes with radius r_0 and small D2-charge p should be (see appendix E):

$$\begin{aligned} dH_3 &= 0, && (34)\\ dG_6 &= J_7, && (35)\end{aligned}$$

$$\begin{aligned} d[Z_1^{-1}(H_3 - g_s *_9 G_6)] &= 0, && (36)\\ d[Z_1^{-1}(*_9 H_3 - g_s G_6)] &= 0, && (37)\end{aligned}$$

where there is a source term J_7 due to D2-charge:

$$J_7 = 2\kappa^2 \tau_{D2} g_s^{-2} p\,\delta(r_1 - r_0)\delta^6(r_2) dr_1 \wedge dr_2 \wedge \omega_5, \tag{38}$$

where $\delta^6(r_2) = \delta(x_4)\delta(x_5)\delta(x_6)\delta(x_7)\delta(x_8)\delta(x_9)$ and $\omega_5 = r_2^5 \cdot \mathrm{dvol}(S^5)$, where $\mathrm{dvol}(S^5)$ denotes the volume-form of an S^5 with unit radius embedded in 456789 subspace and

centered at origin. The source term appears in the Bianchi identity but not in the equations for $Z_1^{-1}(H_3 - g_s *_9 G_6)$ and its 9-d Hodge dual[10].

The equations for $Z_1^{-1}(H_3 - g_s *_9 G_6)$ remain the same as in (6), (7), just with Z replaced by Z_1. Furthermore, since when r goes to infinity, this form is a constant form, we infer that the harmonic form $Z_1^{-1}(H_3 - g_s *_9 G_6) = -\widetilde{\mu}T_3$. Although H_3, G_6, Z_1 all depend on the brane configurations, the combination $Z_1^{-1}(H_3 - g_s *_9 G_6)$ is independent of the brane configurations. This also results in that the potential C_3 is independent of the warp-factor Z_1 and the brane configurations. The difference between equations (4)-(7) and (34)-(37), besides that Z is replaced by a multi-center warp-factor Z_1, is that there is a source term for G_6 since we have introduced D2 sources on this shell.

We can split both H_3 and G_6 into two pieces respectively:

$$
\begin{aligned}
G_6 &= G_6^{(I)} + G_6^{(II)}, && (39)\\
H_3 &= H_3^{(I)} + g_s *_9 G_6^{(II)}, && (40)
\end{aligned}
$$

where $H_3^{(I)}, G_6^{(I)}$ still satisfy the whole four equations (4)-(7) with Z replaced by the multi-center Z_1. In large r region when expanded around r_0, the leading terms of $H_3^{(I)}, G_6^{(I)}$ will reduce to the $n = -7$ solution in section 2. $H_3^{(I)}, G_6^{(I)}$ are the contribution to the fluxes as if there were no D2 source.

The influence of D2 source on the fluxes are mainly on $G_6^{(II)}$, whose equations are now:

$$
dG_6^{(II)} = J_7, \qquad\qquad d *_9 G_6^{(II)} = 0. \tag{41}
$$

The contribution of $G_6^{(II)}$ is dominant over $G_6^{(I)}$ very close to the shell since it has the delta function as source. Since $*_9 G_6^{(II)}$ is closed, it can be written as

$$
*_9 G_6^{(II)} = (r_1^{-2}\partial_1 h dr_1 + r_1^{-2}\partial_2 h dr_2) \wedge \omega_2, \tag{42}
$$

where $\omega_2 = r_1^2 \cdot \mathrm{dvol}(S^2)$, and $\mathrm{dvol}(S^2)$ denotes the volume-form of an S^2 with unit radius embedded in 123 subspace and centered at origin, and $\partial_1 \equiv \frac{\partial}{\partial r_1}$, $\partial_2 \equiv \frac{\partial}{\partial r_2}$. h is a function of r_1, r_2. The solution (see appendix F for more detail) can be expressed through the function defined as $Y = r_1^{-2}\partial_1 h$, and we have

$$
Y = \frac{4\pi C\kappa^2 \tau_{D2} g_s^{-2} p r_0^2}{5 r_1} \partial_{r_0} \left(r_0^{-1} \left[\frac{1}{[(r_1 + r_0)^2 + r_2^2]^{5/2}} - \frac{1}{[(r_1 - r_0)^2 + r_2^2]^{5/2}} \right] \right), \tag{43}
$$

[10]Since in our case the forms H_3 and F_4 both have overall factors of the volume-form of the S^2 in 123 subspace due to the isometry $SO(3) \times SO(6)$, the terms $F_4 \wedge H_3$ and $F_4 \wedge F_4$ are zero and drop off on the right sides of (35), (37).

where the notations of the coefficients in Y are in appendix F.

The leading terms of $G_6^{(II)}$ and $H_3^{(II)} = g_s *_9 G_6^{(II)}$ expanded in terms of r_0, in the region where $r_1 \gg r_0$, is precisely the $n = -9$ solution in section 2, and it is of third order in r_0 and thereby of third order in μ in large r region (see appendix F). The contribution of $G_6^{(II)}$ is dominant over $G_6^{(I)}$ very close to the shell of the brane since it has the delta function as source, while in large r the situation is reversed and $G_6^{(I)}$ becomes dominant over $G_6^{(II)}$ instead. We see the consistency in the calculation of the fluxes in the presence of the sources since the solutions of $H_3^{(I)}, G_6^{(I)}$ and $H_3^{(II)}, G_6^{(II)}$ in this section are the full solutions which just reduce to the $n = -7$ and $n = -9$ solutions in large r region in section 2 respectively.

5 Metric and dilaton in large r region and near each shell

In this section, we come to the discussion on the metric and dilaton. The situation is similar to Polchinski-Strassler [4] that in most regions away from the shells, the D0-charge dominates, and in the regions very close to the shells, the D2-charge dominates instead. This switch of the role of the dominance is also reflected in the change of the dominance between $H_3^{(I)}, G_6^{(I)}$ and $H_3^{(II)}, G_6^{(II)}$ as discussed in last section. We will discuss the metric and dilaton in two limiting regions in this section. One is in the large r region and the other is very near each shell.

The general brane configuration in our solutions are concentric branes with various D0-charge and radii. In the large r region, since the D0-charge dominates, the metric, dilaton and F_2 are very close to the near horizon geometry of multi-center D0 branes with warp-factor Z_1 corresponding to the distributions of these concentric shells of D0 branes. For the general configuration of several concentric shells of S^2-wrapped branes with the ith shell having p_i coincident D2 branes each with q_i units of D0-charge ($N = \sum\limits_i p_i q_i$ and the D0-charge q_i are all large and distribute uniformly on the spheres), the warp factor Z_1 in the solution of the near horizon geometry of multi-center D0 branes, as the superposition of harmonic functions, should be

$$Z_1 = \sum_i \frac{R_i^7}{10 r_1 r_0^{(i)}} \left[\frac{1}{[(r_1 - r_0^{(i)})^2 + r_2^2]^{5/2}} - \frac{1}{[(r_1 + r_0^{(i)})^2 + r_2^2]^{5/2}} \right], \tag{44}$$

where $R_i^7 = 60\pi^3 g_s (p_i q_i) \alpha'^{7/2}$ and $r_0^{(i)} \approx \frac{\pi}{3}\widetilde{\mu}\alpha' q_i$.

Now we will look at the metric and dilaton near each shell of branes, say the ith shell. The total D0-charge of this shell is $N_i = p_i q_i$ and radius of this shell is $r_0^{(i)} \approx \frac{\pi}{3}\widetilde{\mu}\alpha' q_i$. Very close to each shell, the D0-charge no longer have dominant influence over D2-charge since the metric parallel to the shell expand and D0-charge are diluted. We can approximate the metric and dilaton near the S^2, e.g. without loss of generality, near the point $(x_1, x_2, x_3) = (0, 0, r_0^{(i)})$, by the solution of p_i flat D2 branes with B_2 potential on its spatial world-volume [20], [21] (in string frame):

$$
\begin{aligned}
ds^2 &= \frac{\alpha'^{5/2} u_i^{5/2}}{6\pi^2 g_s a_i^{5/2} p_i}\left(-dt^2 + \frac{1}{1+a_i^5 u_i^5}(d\widetilde{x}_{i1}^2 + d\widetilde{x}_{i2}^2)\right) + \frac{6\pi^2 g_s a_i^{5/2} p_i}{\alpha'^{1/2} u_i^{5/2}}(du_i^2 + u_i^2 d\Omega_6^2), \\
e^{2\Phi} &= \frac{(6\pi^2)^3 g_s^5 a_i^{15/2} p_i^3}{\alpha'^{15/2} u_i^{15/2}}\left(\frac{a_i^5 u_i^5}{1+a_i^5 u_i^5}\right), \qquad (45)
\end{aligned}
$$

where the label i denotes the ith shell. $\widetilde{x}_{i1}$,$\widetilde{x}_{i2}$ parameterize the spatial part of the D2 branes, u_i is the energy direction away from the ith shell of branes in the transverse direction and a_i is a constant that will be worked out in (49).

Since there is B_2 field on the D2 branes in solution (45), the D2 branes can couple to C_1 and there is D0-charge on the D2 branes. Suppose we are looking at regions only near the ith shell but not the other shells at the same time. When away from this shell of branes such that $a_i^5 u_i^5 \gg 1$, i.e. $\frac{a_i^5 u_i^5}{1+a_i^5 u_i^5} \approx 1$, the above metric and dilaton (45) match exactly with the near horizon geometry of multi-center D0 branes, near this shell of branes (x_l: x_1, x_2, x_3 and x_a: $x_4, ..., x_9$):

$$
\begin{aligned}
ds^2 &= -\frac{\sqrt{10} r_0^{(i)} \rho_i^{5/2}}{R_i^{7/2}} dt^2 + \frac{R_i^{7/2}}{\sqrt{10} r_0^{(i)} \rho_i^{5/2}}(d\overrightarrow{x}_l^2 + d\overrightarrow{x}_a^2), \\
e^{2\Phi} &= g_s^2 Z_1^{3/2} = \frac{g_s^2 R_i^{21/2}}{10^{3/2}\left(r_0^{(i)}\right)^3 \rho_i^{15/2}}, \qquad (46)
\end{aligned}
$$

where $r_0^{(i)} \approx \frac{\pi}{3}\widetilde{\mu}\alpha' q_i$, $R_i^7 = 60\pi^3 g_s (p_i q_i)\alpha'^{7/2}$. We used the multi-center warp-factor Z_1 in (44) approximated near the ith shell of the brane source, and ρ_i is the distance away from the ith shell:

$$
Z_1 \approx \frac{R_i^7}{10\left(r_0^{(i)}\right)^2 \rho_i^5}, \qquad (47)
$$

$$
\rho_i = [(r_1 - r_0^{(i)})^2 + r_2^2]^{1/2}. \qquad (48)
$$

We can define a cross-over distance $\rho_c^{(i)}$ as the distance away from the brane where $a_i u_i = 1$, which could characterize the regions of influence of the D2-charge. In order for the match to be valid, we need when away from the shell the two approximations $\frac{a_i^5 u_i^5}{1+a_i^5 u_i^5} \approx 1$, (i.e. $\rho_i \gg \rho_c^{(i)}$) and $Z_1 \approx \frac{R_i^7}{10\left(r_0^{(i)}\right)^2 \rho_i^5}$, (i.e. $\rho_i \ll r_0^{(i)}$) are both valid. This requires the match to happen in the region $\rho_c^{(i)} \ll \rho_i \ll r_0^{(i)}$, so for our approximation to be valid we need the parameters satisfy $\rho_c^{(i)} \ll r_0^{(i)}$.

Under this approximation, the dilaton and metric match exactly and we thereby found the relation between the parameters and variables by comparing the two solutions (45), (46):

$$\begin{aligned} u_i &= \frac{\rho_i}{\alpha'}, \qquad a_i = \left(\frac{R_i^{7/2}}{\sqrt{10} r_0^{(i)} \cdot 6\pi^2 g_s p_i}\right)^{2/5}, \qquad \rho_c^{(i)} = \alpha' \left(\frac{\sqrt{10} r_0^{(i)} \cdot 6\pi^2 g_s p_i}{R_i^{7/2}}\right)^{2/5}, \\ \frac{\widetilde{x}_{i1,i2}}{x_{1,2}} &= \frac{R_i^7}{10\left(r_0^{(i)}\right)^2 \alpha'^{5/2} \cdot 6\pi^2 g_s p_i}. \end{aligned} \tag{49}$$

So the metric and dilaton near the ith shell can be written as

$$\begin{aligned} ds^2 &= -\frac{\sqrt{10} r_0^{(i)} \rho_i^{5/2}}{R_i^{7/2}} dt^2 + \frac{R_i^{7/2}}{\sqrt{10} r_0^{(i)} \rho_i^{5/2}} (dx_3^2 + d\vec{x}_a^{\,2}) + \frac{R_i^{7/2} \rho_i^{5/2}}{\sqrt{10} r_0^{(i)} \left(\rho_i^5 + \left(\rho_c^{(i)}\right)^5\right)} (dx_1^2 + dx_2^2), \\ e^{2\Phi} &= \frac{g_s^2 R_i^{21/2}}{10^{3/2} \left(r_0^{(i)}\right)^3 \rho_i^{5/2} \left(\rho_i^5 + \left(\rho_c^{(i)}\right)^5\right)}, \end{aligned} \tag{50}$$

and as discussed above it is valid when

$$\frac{r_0^{(i)}}{\rho_c^{(i)}} \sim \left(\frac{q_i^5}{g_s N_i}\right)^{1/5} \gg 1, \tag{51}$$

which is just $\frac{q_i^5}{g_s N_i} \gg 1$, where q_i is the D0-charge of each D2 brane on the ith shell and N_i is the total D0-charge of the ith shell. This is the same scaling bound condition as (30) in section 3.

For general brane configurations of several shells, the metric and dilaton in complete regions are very difficult to solve explicitly, but it's clear that in large r region they approach the near horizon geometry of multi-center D0 branes with warp-factor Z_1 in (44), and near each shell they are approximated as the solutions in (50). In special cases when there is only one single shell of branes, the metric and dilaton may be expressed

approximately valid in all regions. Suppose there are p D2 branes each with D0-charge q on this single shell. $pq = N$ is the total D0 charge and the radius is $r_0 \approx \frac{\pi}{3}\widetilde{\mu}\alpha' q$. The solution near the brane can be obtained from (50) by identifying $p_i = p, q_i = q$. And then we can generalize the solution near the shell to all regions by replacing the warp-factor $Z_1 \approx \frac{R^7}{10r_0^2\rho^5}$, approximated near the shell as in (47), with the warp-factor Z_1 in all regions as in (33). The validity of this approximation is again from (51), i.e. $\frac{q^5}{g_s N} \gg 1$.

6 Related issues and generalizations to other plane-wave M(atrix) theories

So far we have constructed the supergravity solutions dual to the 1/2 BPS concentric fuzzy sphere vacua of the BMN matrix model using the method of the Polchinski-Strassler solution, which is the string dual of the $\mathcal{N} = 1^*$ theory. In this last section, we discuss some related issues or remaining issues to our discussions above.

Each 1/2 BPS vacuum of BMN matrix model can be represented by a Young tableau [13], [9] and it can be interpreted as either concentric D2 branes or concentric NS5 branes, in different regimes of parameters [8], [13]. The configurations in terms of concentric shells of D2 branes have their validities as dual descriptions when the effective 't Hooft coupling in the matrix perturbation theory of the BMN matrix model when expanding around each fuzzy sphere is small[11] [8], [13]. When the effective 't Hooft couplings are small, the interactions are smaller than the harmonic oscillator energies expanded around these fuzzy spheres, and the BMN matrix model can be studied perturbatively around these fuzzy spheres [8]. The concentric D2 brane configurations are therefore good descriptions of the BMN vacua when these parameters are small. For fixed partition of N, we can always tune μ and R to satisfy these conditions. For fixed μ and R, it is relatively safer to expand around a fuzzy sphere when the numbers of the coincident spheres are smaller and the matrix size of the sphere is larger [8].

We have analyzed the descriptions of the vacua in terms of concentric D2 branes in the regime of weak effective 't Hooft couplings and we haven't studied the situation of the concentric NS5 branes in detail, which are expected to be valid in the regime of strong

[11] The effective 't Hooft coupling is $p_i\left(\frac{1}{\mu p_+^{(i)}}\right)^3$ [8], where $p_+^{(i)} = \frac{q_i}{R}$ is the light-cone momentum of the M2 brane on the ith shell and p_i is the number of coincident M2 branes on the ith shell.

effective 't Hooft couplings. The NS5 branes polarized from D0 branes should also have equilibrium radii with these additional fluxes turned on since it can couple to the dual of the NS-NS 3-form flux and the 3-form potential via world-volume 3-form flux.

In fact, there could be smooth solutions that are dual to these vacua. The smooth solutions and the solutions with brane configurations studied in this paper may be related by geometric transitions (e.g. [32]), where branes and fluxes get replaced with each other.

There are less supersymmetric vacua and time-dependent vacua e.g. [8], [10], [33], [34], [35], [37] that we have not discussed. For example, it would be interesting to understand such as the 1/4 BPS rotating elliptic fuzzy spheres described by [34], and the 1/4 BPS rotating fuzzy spheres by [36]. There are also instanton solutions [37] which are similar to the domain wall solutions in the $\mathcal{N} = 1^*$ theory [38]. It would be good to understand the dual descriptions of the vacua in the model that is less supersymmetric and/or non-static.

The Polchinski-Strassler type solution has been widely generalized and applied to many other situations in terms of other branes (e.g. [39], [40], [41], [42]). One can conjecture that the Polchinski-Strassler type solution is universal for any Dp branes [31], which can be polarized to Dp+2 or NS5 branes in the presence of the additional R-R and NS-NS fluxes on the background of the near horizon geometry of N Dp branes and the resulting solutions are dual to the mass-deformed world-volume field theory of N Dp branes. In each such solution, we have a pair of R-R and NS-NS fluxes and this is mainly because we have two channels of polarizations. The R-R flux is more responsible for polarization to Dp+2 branes, while the NS-NS flux is more responsible for polarization to NS5 branes.

The construction of dual supergravity descriptions to the BMN matrix model may be generalized to those of the M(atrix) theories on less supersymmetric plane-wave backgrounds [43], [44]. For a general plane-wave matrix theory with lagrangian, e.g. [43], [25]:

$$L = \frac{1}{2}\mathrm{Tr}\{\sum_i (D_0 X^i)^2 + \sum_{i,j}\frac{1}{2}[X^i, X^j]^2 + i\psi^T D_0\psi - \psi^T\gamma_i[X^i,\psi]$$
$$-\sum_i \mu_i^2\left(X^i\right)^2 + \frac{2}{3}i\widetilde{T}_{ijk}X^iX^jX^k - \frac{1}{4}i\psi^T\,\widetilde{\not{T}}\psi\}, \tag{52}$$

where $\widetilde{\not{T}}=\frac{1}{3!}\widetilde{T}_{ijk}\gamma^{ijk}$, $\sum\limits_i \mu_i^2 = \sum\limits_{i,j,k}\frac{1}{12}(\widetilde{T}_{ijk})^2$, there might exist supergravity duals which are similar to the case of the BMN matrix model. In the large r region, the dual supergravity solutions can also be considered as perturbations around the near horizon geometry of N D0 branes by the additional fluxes H_3 and G_6, which should also satisfy equations (4)-

(7), and the form $Z^{-1}(H_3 - g_s *_9 G_6)$, where $Z = \frac{R^7}{r^7}$, should thereby also be annihilated by both d and $d*_9$ in the transverse 9-d. When r goes to infinity it should approach a constant form, so the additional fluxes should satisfy the relation in large r region as:

$$Z^{-1}(H_3 - g_s *_9 G_6) \propto \widetilde{T}_3, \tag{53}$$

where $\widetilde{T}_3 = \frac{1}{3!}\widetilde{T}_{ijk} dx^i \wedge dx^j \wedge dx^k$, and $\widetilde{T}_{ijk}$ are the coefficients of the Myers term in the corresponding M(atrix) theory on the general plane-wave background. Thereby one can conjecture that the perturbation by these mass terms and Myers terms are dual to turning on these additional fluxes H_3 and G_6 causing D0 branes to polarize into some non-spherical branes. The brane configuration would be more difficult to describe than the case of the BMN matrix model. For example, in the M(atrix) theory on the background of T-dual of the IIB pp-wave lifted to 11d, the BPS vacua correspond to M2 branes polarized into M5 brane, where the M2 branes distributed on a fuzzy ellipsoid [44]. So generally, the brane in the small r region would take the shape that corresponds to the classical static solution in the corresponding M(atrix) theory and it should also equivalently be the shape of a probe brane in the presence of external fluxes H_3 and G_6.

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A IIA equations for fluxes

The IIA equations of motion in this appendix are in Einstein frame and in the convention of [45]. The bosonic equations of motion are:

$$\begin{aligned} d * d(\Phi) &= -\frac{1}{2} g_s e^{-\Phi} H_3 \wedge *H_3 + \frac{3}{4} g_s^{1/2} e^{3\Phi/2} F_2 \wedge *F_2 + \frac{1}{4} g_s^{3/2} e^{\Phi/2} \widetilde{F}_4 \wedge *\widetilde{F}_4, & (54) \\ d(e^{3\Phi/2} * F_2) &= g_s e^{\Phi/2} H_3 \wedge *\widetilde{F}_4, & (55) \\ d(e^{\Phi/2} * \widetilde{F}_4) &= -g_s^{1/2} F_4 \wedge H_3, & (56) \\ \frac{1}{2} g_s F_4 \wedge F_4 &= d(e^{-\Phi} * H_3 + g_s^{1/2} e^{\Phi/2} C_1 \wedge *\widetilde{F}_4), & (57) \end{aligned}$$

where $F_2 = dC_1, F_4 = dC_3, H_3 = dB_2, \widetilde{F}_4 = F_4 - C_1 \wedge H_3$, and the Bianchi identities are: $dF_2 = 0, dF_4 = 0, dH_3 = 0$. The relation between the Einstein frame metric and string frame metric is $(G_{\mu\nu})_{\text{Einstein}} = g_s^{1/2} e^{-\Phi/2} (G_{\mu\nu})_{\text{string}}$.

The unperturbed background is the near horizon geometry of N D0 branes. In Einstein frame it is

$$
\begin{aligned}
ds^2 &= -Z^{-7/8} dt^2 + Z^{1/8} d\overrightarrow{x}_i^2, \qquad i = 1, ..., 9, \\
e^{\Phi} &= g_s Z^{3/4}, \qquad C_1 = g_s^{-1}(Z^{-1} - 1) dt, \qquad Z = \frac{R^7}{r^7}.
\end{aligned} \tag{58}
$$

The perturbed fluxes H_3 and $\widetilde{F}_4$ are small fluctuations in large r region. The perturbed terms in the right sides of equation (54), (55), if non-zero, are at least of second order fluctuations. Therefore if neglecting the terms of second or higher orders in the fluctuations, the equations for the dialton and F_2 still have the form:

$$
d * d(\Phi) = \frac{3}{4} g_s^{1/2} e^{3\Phi/2} F_2 \wedge *F_2, \qquad d(e^{3\Phi/2} * F_2) = 0, \qquad dF_2 = 0. \tag{59}
$$

And therefore the other fluxes H_3 ,$\widetilde{F}_4$ obey:

$$
d(Z^{3/8} * \widetilde{F}_4) = 0, \qquad dH_3 = 0. \tag{60}
$$

$$
dF_4 = 0, \qquad d(g_s^{-1} Z^{-3/4} * H_3 + g_s Z^{3/8} C_1 \wedge *\widetilde{F}_4) = 0. \tag{61}
$$

From (60) we can define $G_6 = Z^{3/8} * \widetilde{F}_4$, so that $dG_6 = 0$. We can write (61) in terms of the 9-d Hodge dual $*_9$ in the transverse 9-d with respect to the flat metric: $*H_3 = dx^0 \wedge (-Z^{-1/4} *_9 H_3)$, $\widetilde{F_4} = dx^0 \wedge (-Z^{-1} *_9 G_6)$, then from (61) we precisely arrive at equation (6), (7) in section 2.

B Linearized solutions of fluxes

In this appendix we discuss linearized solutions of additional fluxes in terms of tensor harmonics. Without loss of generality, we can define a T_3 analogous to Polchinski-Strassler

[4] as follows:

$$
\begin{aligned}
T_3 &= dr_1 \wedge \omega_2 = dx^1 \wedge dx^2 \wedge dx^3, & (62)\\
S_2 &= r_1\omega_2 = \frac{1}{2!}\varepsilon_{ijk}x^i \wedge dx^j \wedge dx^k, \qquad dS_2 = 3T_3, & (63)\\
V_3 &= d\ln r \wedge S_2 = \frac{r_1^2}{r^2}dr_1 \wedge \omega_2 + \frac{r_1 r_2}{r^2}dr_2 \wedge \omega_2 & (64)\\
&= \frac{r_1^2}{r^2}dx^1 \wedge dx^2 \wedge dx^3 + \frac{1}{2!}\frac{x^a x^i}{r^2}\varepsilon_{ijk}dx^a \wedge dx^j \wedge dx^k, & (65)\\
(i,j,k &= 1,2,3, a = 4, ..., 9)
\end{aligned}
$$

where the definition of ω_2, ω_5 are in section 4. Then

$$
\begin{aligned}
*_9\, T_3 &= dr_2 \wedge \omega_5, & (66)\\
*_9 V_3 &= \frac{r_1^2}{r^2}dr_2 \wedge \omega_5 - \frac{r_1 r_2}{r^2}dr_1 \wedge \omega_5. & (67)
\end{aligned}
$$

If we search for the solutions of the form in large r region according to the isometry $SO(3) \times SO(6)$:

$$H_3 = r^m(\alpha T_3 + \beta V_3), \qquad G_6 = g_s^{-1} r^n(\gamma *_9 T_3 + \delta *_9 V_3), \tag{68}$$

where $m, n, \alpha, \beta, \gamma, \delta$ are constants. The g_s^{-1} factor in the ansatz for G_6 is introduced for convenience in this appendix. The four equations (4), (5), (6), (7) give the constraints:

$$dH_3 = 0, \quad \Rightarrow \quad \beta = \frac{m}{3}\alpha. \tag{69}$$

$$dG_6 = 0, \quad \Rightarrow \quad \delta = -\frac{n}{n+6}\gamma. \tag{70}$$

$$d[Z^{-1}(H_3 - g_s *_9 G_6)] = 0, \quad \Rightarrow \quad 7\alpha r^m - \frac{n^2+16n+42}{n+6}\gamma r^n = 0. \tag{71}$$

$$d[Z^{-1}(*_9 H_3 - g_s G_6)] = 0, \quad \Rightarrow \quad \frac{m^2+16m+21}{3}\alpha r^m - \frac{42}{n+6}\gamma r^n = 0. \tag{72}$$

Since in the perturbation, α, γ cannot be both zero, we need m, n to be equal, and then we get

$$n(n+16)(n+7)(n+9) = 0. \tag{73}$$

This leads to 4 solutions that are two pairs of non-normalizable and normalizable solutions in (10), (11), (12), (13) in section 2.

C Killing spinors

The IIA susy transformation rules we used are in string frame (e.g. [28]):

$$\delta\psi_m = [(\partial_m + \frac{1}{4}\omega_{mab}\Gamma^{ab} + \frac{1}{8}e^{\Phi}\ \not{F}_2\Gamma_m\Gamma^{\underline{11}}) + (\frac{1}{8}H_{mab}\Gamma^{ab}\Gamma^{\underline{11}} + \frac{1}{8}e^{\Phi}\widetilde{\not{F}}_4\Gamma_m)]\epsilon, \quad (74)$$

$$\delta\lambda = [(\frac{1}{2}\Gamma^m\partial_m\Phi + \frac{3}{8}e^{\Phi}\ \not{F}_2\Gamma^{\underline{11}}) + (\frac{1}{4}\ \not{H}_3\Gamma^{\underline{11}} + \frac{1}{8}e^{\Phi}\widetilde{\not{F}}_4)]\epsilon, \quad (75)$$

where letters with a slash denote the contractions of forms with gamma matrices: $\not{F}_2 = \frac{1}{2!}F_{ab}\Gamma^{ab}$, $\not{H}_3 = \frac{1}{3!}H_{abc}\Gamma^{abc}$, $\widetilde{\not{F}}_4 = \frac{1}{4!}\widetilde{F}_{abcd}\Gamma^{abcd}$ and similarly for other forms.

The unperturbed Killing spinors $\epsilon^{(0)}$ in the absence of H_3, G_6 satisfy:

$$[\frac{1}{2}\Gamma^m\partial_m\Phi + \frac{3}{8}e^{\Phi}\ \not{F}_2\Gamma^{\underline{11}}]\epsilon^{(0)} = \frac{3}{8}Z^{-5/4}\Gamma^{\underline{i}}\partial_i Z(1+\Gamma^{\underline{0}}\Gamma^{\underline{11}})\epsilon^{(0)} = 0, \quad (76)$$

$$[\partial_0 + \frac{1}{4}\omega_{0ab}\Gamma^{ab} + \frac{1}{8}e^{\Phi}\ \not{F}_2\Gamma_0\Gamma^{\underline{11}}]\epsilon^{(0)} = [\partial_0 + \frac{1}{8}Z^{-3/2}\Gamma^{\underline{0}}\Gamma^{\underline{i}}\partial_i Z(1+\Gamma^{\underline{0}}\Gamma^{\underline{11}})]\epsilon^{(0)} = 0, \quad (77)$$

$$\begin{aligned}[\partial_i + \frac{1}{4}\omega_{iab}\Gamma^{ab} + \frac{1}{8}e^{\Phi}\ \not{F}_2\Gamma_i\Gamma^{\underline{11}}]\epsilon^{(0)} &= [(\partial_i + \frac{1}{8}Z^{-1}\partial_i Z) - \frac{1}{8}Z^{-1}\partial_i Z(1+\Gamma^{\underline{0}}\Gamma^{\underline{11}}) \\ &\quad + \frac{1}{8}Z^{-1}\Gamma^{\underline{i}}\Gamma^{\underline{j}}\partial_j Z(1+\Gamma^{\underline{0}}\Gamma^{\underline{11}})]\epsilon^{(0)} = 0.\ (j \neq i), \quad (78)\end{aligned}$$

where the indices i, j denotes 1,...,9 and the indices of gamma matrices with a bar below are the gamma matrices in tangent space. So we have the already familiar result (e.g. [29], [30]):

$$\epsilon^{(0)} = Z^{-1/8}\eta, \quad (79)$$

where η is a constant spinor satisfying $(1+\Gamma^{\underline{0}}\Gamma^{\underline{11}})\eta = 0$. The Killing spinors in the presence of small fluctuations of H_3, G_6 can be written as

$$\epsilon = \epsilon^{(0)} + \epsilon^{(1)}, \quad (80)$$

where $\epsilon^{(1)}$ is the perturbation around $\epsilon^{(0)}$. When we turn on the H_3, G_6 that are of first order in μ, $\epsilon^{(1)}$ is of order μ. The variations of gravitino and dilatino of the first order in μ give:

$$\frac{3}{8}Z^{-5/4}\Gamma^{\underline{i}}\partial_i Z(1+\Gamma^{\underline{0}}\Gamma^{\underline{11}})\epsilon^{(1)} = -[\frac{1}{4}\ \not{H}_3\Gamma^{\underline{11}} + \frac{1}{8}e^{\Phi}\widetilde{\not{F}}_4]\epsilon^{(0)}, \quad (81)$$

$$[\partial_0 + \frac{1}{8}Z^{-3/2}\Gamma^{\underline{0}}\Gamma^{\underline{i}}\partial_i Z(1+\Gamma^{\underline{0}}\Gamma^{\underline{11}})]\epsilon^{(1)} = -[\frac{1}{8}H_{0ab}\Gamma^{ab}\Gamma^{\underline{11}} + \frac{1}{8}e^{\Phi}\widetilde{\not{F}}_4\Gamma_0]\epsilon^{(0)}, \quad (82)$$

$$\begin{aligned}&[(\partial_i + \frac{1}{8}Z^{-1}\partial_i Z) - \frac{1}{8}Z^{-1}\partial_i Z(1+\Gamma^{\underline{0}}\Gamma^{\underline{11}}) + \frac{1}{8}Z^{-1}\Gamma^{\underline{i}}\Gamma^{\underline{j}}\partial_j Z(1+\Gamma^{\underline{0}}\Gamma^{\underline{11}})]\epsilon^{(1)} \\ &= -[\frac{1}{8}H_{iab}\Gamma^{ab}\Gamma^{\underline{11}} + \frac{1}{8}e^{\Phi}\widetilde{\not{F}}_4\Gamma_i]\epsilon^{(0)}.\ (j \neq i.) \quad (83)\end{aligned}$$

Now we can first consider the dilatino variation (81) involving $\epsilon^{(1)}$. Since $\epsilon^{(0)}$ is not time-dependent while $\epsilon^{(1)}$ is time-dependent, the time-dependent part of $\epsilon^{(1)}$ should be annihilated by $(1+\Gamma^{\underline{0}}\Gamma^{\underline{11}})$. So we can decompose $\epsilon^{(1)}$ into two parts: $\epsilon^{(1)}=\epsilon_1^{(1)}+\epsilon_2^{(1)}$, where $\epsilon_1^{(1)}$ is time-dependent and $\epsilon_2^{(1)}$ is not time-dependent.

We can then split dilatino equation (81) into two equations:

$$\begin{aligned}(1+\Gamma^{\underline{0}}\Gamma^{\underline{11}})\epsilon_1^{(1)} &= 0, &(84)\\ \Gamma^{\underline{i}}\partial_i Z(1+\Gamma^{\underline{0}}\Gamma^{\underline{11}})\epsilon_2^{(1)} &= -\frac{8}{3}Z^{5/4}[\frac{1}{4}\not{H}_3\Gamma^{\underline{11}}+\frac{1}{8}e^{\Phi}\widetilde{\not{F}}_4]\epsilon^{(0)}. &(85)\end{aligned}$$

Substituting (84), (85) into the time-component of the gravitino variation, equation (82), and the spatial components of gravitino variation, equation (83), we have:

$$\partial_0\epsilon_1^{(1)}=\frac{1}{12}Z^{-1/4}\Gamma^{\underline{0}}[\not{H}_3\Gamma^{\underline{11}}-e^{\Phi}\widetilde{\not{F}}_4]\epsilon^{(0)}, \qquad (\partial_i+\frac{1}{8}Z^{-1}\partial_i Z)\epsilon_1^{(1)}=0. \qquad (86)$$

Since the right side of the first equation in (86) is time-independent, we solve that $\epsilon_1^{(1)}$ is linear in time:

$$\epsilon_1^{(1)}=\frac{1}{12}\cdot\frac{1}{3!}Z^{-1}[H_{ijk}-g_s(*_9G_6)_{ijk}]\Gamma^{\underline{ijk}}\epsilon^{(0)}t, \qquad (87)$$

and $\epsilon_1^{(1)}$ has the same helicity to $\epsilon^{(0)}$. The spatial-independence of $Z^{-1/8}\epsilon_1^{(1)}$ from the second equation in (86) and (79) imply:

$$Z^{-1}[H_{ijk}-g_s(*_9G_6)_{ijk}]=\text{const}. \qquad (88)$$

The discussion so far doesn't require $SO(3)\times SO(6)$ symmetry but only that the fluxes H_3 and G_6 be small. There are stronger constraints than merely that $Z^{-1}[H_3-g_s*_9G_6]$ would be a constant from the spatial part of the gravitino variation involving $\epsilon_2^{(1)}$ from (83):

$$\begin{aligned}&[(\partial_i+\frac{1}{8}Z^{-1}\partial_i Z)-\frac{1}{8}Z^{-1}\partial_i Z(1+\Gamma^{\underline{0}}\Gamma^{\underline{11}})+\frac{1}{8}Z^{-1}\Gamma^{\underline{i}}\Gamma^{\underline{j}}\partial_j Z(1+\Gamma^{\underline{0}}\Gamma^{\underline{11}})]\epsilon_2^{(1)}\\ =\;&[-\frac{1}{8}H_{iab}\Gamma^{ab}+\frac{1}{8}Z^{1/4}\not{G}_3\Gamma^{\underline{i}}]\Gamma^{\underline{0}}\epsilon^{(0)}, \qquad (89)\end{aligned}$$

where we define $G_3=g_s*_9G_6$. If contracting both sides of equation (89) with $\Gamma^{\underline{i}}$, and input equation (85), and then acting on both sides the projection $(1-\Gamma^{\underline{0}}\Gamma^{\underline{11}})$, the right side becomes zero and we have

$$(1-\Gamma^{\underline{0}}\Gamma^{\underline{11}})\epsilon_2^{(1)}=0. \qquad (90)$$

So $\epsilon_2^{(1)}$ has the opposite chirality with respect to $\epsilon^{(0)}$ and $\epsilon_1^{(1)}$.

From now on, we will use the indices l, m, n to denote 1,2,3,4,...,9, the indices i, j, k to denote 1,2,3, and the indices a, b, c to denote 4,...,9, for convenience. Applying the projection condition (90) on $\epsilon_2^{(1)}$ and then equation (85) becomes:

$$\Gamma^{\underline{l}}\partial_l Z\epsilon_2^{(1)} = -\frac{4}{3}Z^{5/4}\left[\frac{1}{4}\not{H}_3 + \frac{1}{8}\not{G}_3\right]\Gamma^{\underline{0}}\epsilon^{(0)}. \tag{91}$$

Substituting (91) into (89) and using the projection condition (90), we have 9 individual spatial equations:

$$\partial_l[Z^{-3/8}\epsilon_2^{(1)}] = Z^{-1/4}\left[\frac{1}{12}\Gamma^{\underline{l}}\not{H}_3 - \frac{1}{8}H_{\underline{lmn}}\Gamma^{\underline{mn}} + \frac{1}{24}\Gamma^{\underline{l}}\not{G}_3 + \frac{1}{8}\not{G}_3\Gamma^{\underline{l}}\right]\Gamma^{\underline{0}}\eta. \tag{92}$$

Now if the we look at the fluxes in the ansatz (8), (9) and combine the first supersymmetry condition from (88), we have

$$H_3 = R^7r^{-7}(-\alpha T_3 - \beta V_3), \qquad G_3 = R^7r^{-7}(\gamma T_3 - \beta V_3), \tag{93}$$

where α, β, γ are constants.

By dimensional analysis from equation (91), (92), one finds that $\epsilon_2^{(1)}$ after extracted out the $Z^{3/8}$ factor should be linear in x^l so we can try the ansatz:

$$\epsilon_2^{(1)} = (\mu_1\Gamma^{\underline{i}}x^i + \mu_4\Gamma^{\underline{a}}x^a)Z^{3/8}\Gamma^{\underline{123}}\Gamma^{\underline{0}}\eta, \tag{94}$$

where μ_1, μ_4 are constants. Then comparing the left and right side of equation (91), we get the relation

$$\mu_1 = \frac{-2\alpha + \gamma}{42} - \frac{\beta}{14}, \qquad \mu_4 = \frac{-2\alpha + \gamma}{42}. \tag{95}$$

Comparing the left and right side of equation (92), we get another relation

$$\mu_1 = \frac{\alpha + \gamma}{6}, \qquad \mu_4 = -\frac{\alpha + \gamma}{12}. \tag{96}$$

This shows $\mu_1 : \mu_4 = 2 : -1$, and combine with (95) and (96) we have $\alpha : \beta : \gamma = 3 : -7 : -1$, which is just our $n = -7$ solution (22) in section 2.

D Approximation of D2 potential

In this appendix, we write some details in the approximation of D2 potential in section 3. The DBI and WZ action of the a D2 brane with q units of D0 charge in the string frame is in (23)

$$S_{D2} = -\tau_{D2}\int d^3\sigma e^{-\Phi}\sqrt{-\det(G_{\alpha\beta} + 2\pi\alpha'\mathcal{F}_{\alpha\beta})} + \tau_{D2}\int(C_3 + 2\pi\alpha'\mathcal{F}_2 \wedge C_1), \tag{97}$$

where $2\pi\alpha^{'}\mathcal{F}_2 = 2\pi\alpha^{'}F_2 - B_2$. We choose the gauge that the world-volume coordinates are the same as the space-time ones, i.e. t, θ, φ, where the angles parameterize the S^2. $G_{\shortparallel}, G_{\perp}$ are the pull-back metrics parallel to the time and the spherical directions respectively, so we have:

$$\det G_{\shortparallel} = -Z^{-1/2}, \qquad \det G_{\perp} = Z r_1^4 \sin^2\theta, \tag{98}$$

where r_1 is the radius in 123 subspace.

The D0-charge of the D2 brane is q, so the world-volume 2-form fluxes F_2 is:

$$F_2 = \frac{1}{2} q \sin\theta d\theta \wedge d\varphi, \qquad \int_{S^2} F_2 = 2\pi q, \tag{99}$$

so we have $F_{\theta\varphi} = \frac{1}{2} q \sin\theta$, $\det F_2 = \frac{1}{2} F_{\alpha\beta}F^{\alpha\beta} \det G_{\perp} = \frac{q^2}{4}\sin^2\theta$, $F_{\alpha\beta}F^{\alpha\beta} = \frac{q^2}{2Zr_1^4}$.

Suppose the dominating terms in both DBI and WZ actions are from the contribution of F_2, which requires the conditions (24) then we can expand the square-root in the DBI action as

$$\sqrt{-\det(G_{\alpha\beta} + 2\pi\alpha'\mathcal{F}_{\alpha\beta})} \approx \sqrt{-\det G_{\shortparallel}} \left(2\pi\alpha^{'}\sqrt{\det F_2} + \frac{\det G_{\perp}}{4\pi\alpha'\sqrt{\det F_2}} \right). \tag{100}$$

The leading term in the DBI part is

$$-\tau_{D2} g_s^{-1} \int d^3\sigma Z^{-3/4} \sqrt{-\det G_{\shortparallel}} \cdot 2\pi\alpha^{'}\sqrt{\det F_2} = -\int dt d\theta d\varphi \left(2\pi\alpha^{'}\tau_{D2} g_s^{-1} Z^{-1} \cdot \frac{1}{2} q \sin\theta \right). \tag{101}$$

The leading term in the WZ part is

$$\tau_{D2} \int 2\pi\alpha^{'} F_2 \wedge C_1 = \int dt d\theta d\varphi \left(2\pi\alpha^{'}\tau_{D2} g_s^{-1} Z^{-1} \cdot \frac{1}{2} q \sin\theta \right), \tag{102}$$

where we choose the gauge choice $C_1 = g_s^{-1} Z^{-1} dx^0$ in section 3. The two leading terms from the DBI part and WZ part precisely cancel.

The subleading terms of the DBI and WZ parts read respectively:

$$-\tau_{D2} g_s^{-1} \int d^3\sigma Z^{-3/4} \frac{\sqrt{-\det G_{\shortparallel}}\det G_{\perp}}{4\pi\alpha'\sqrt{\det F_2}} = -\int dt d\theta d\varphi \left(\tau_{D2} g_s^{-1} \frac{r_1^4 \sin\theta}{2\pi\alpha' q} \right) = \int dt \left(-\frac{2\tau_{D2} r_1^4}{g_s \alpha' q} \right), \tag{103}$$

where the Z factor cancel exactly in (103) and

$$\tau_{D2} \int C_3 = \int dt d\theta d\varphi \left(\frac{1}{3} \tau_{D2} g_s^{-1} \widetilde{\mu} r_1^3 \sin\theta \right) = \int dt \left(\frac{4\pi\tau_{D2}\widetilde{\mu} r_1^3}{3 g_s} \right), \tag{104}$$

where $C_3 = \frac{1}{3} g_s^{-1} \widetilde{\mu} dx^0 \wedge S_2$ since $F_4 = \widetilde{F}_4 + C_1 \wedge H_3 = -g_s^{-1}\widetilde{\mu} dx^0 \wedge T_3$.

E Putting source

The IIA bosonic action without external source is [45]

$$\begin{aligned} S_{IIA} &= \frac{1}{2\kappa^2}[\int d^{10}x\sqrt{-G}R - \frac{1}{2}\int(d\Phi\wedge *d\Phi + g_s e^{-\Phi}H_3\wedge *H_3 + g_s^{1/2}e^{3\Phi/2}F_2\wedge *F_2 \\ &\quad + g_s^{3/2}e^{\Phi/2}\widetilde{F}_4\wedge *\widetilde{F}_4 + g_s^2 B_2\wedge F_4\wedge F_4)]. \end{aligned} \tag{105}$$

When we add D2 branes with D0-charge distributed in a single shell as in section 4, we actually introduced the source terms in the action. In analogy to electricity and magnetism, now in the action there should appear a source term (some related discussion on adding source term is in [46]):

$$S_{source} = \frac{1}{2\kappa^2}\int g_s^2(C_3 - B_2\wedge C_1)\wedge J_7, \tag{106}$$

where $J_7 = 2\kappa^2\tau_{D2}g_s^{-2}p\delta(r_1-r_0)\delta^6(r_2)dr_1\wedge dr_2\wedge\omega_5$ and $\delta^6(r_2) = \delta(x_4)\delta(x_5)\delta(x_6)\delta(x_7)\delta(x_8)\delta(x_9)$. The coefficient in J_7 are read off from comparing the WZ action of the D2 branes with D0-charge involving the couplings to C_3, B_2. So now the total action is

$$S = S_{IIA} + S_{source}. \tag{107}$$

Since now C_3 couples to J_7, and B_2 couples to $\frac{1}{2}F_4\wedge F_4 + C_1\wedge J_7$, the equations for $d(e^{\Phi/2} * \widetilde{F}_4)$ and for $d(e^{-\Phi} * H_3)$ when adding source should be modified to

$$\begin{aligned} d(e^{\Phi/2} * \widetilde{F}_4) &= -g_s^{1/2}F_4\wedge H_3 + g_s^{1/2}J_7, &(108)\\ d(e^{-\Phi} * H_3 + g_s^{1/2}e^{\Phi/2}C_1\wedge *\widetilde{F}_4) &= \frac{1}{2}g_s F_4\wedge F_4 + g_s C_1\wedge J_7. &(109) \end{aligned}$$

In our cases, since the forms H_3 and F_4 both have overall factors of the volume-form of the S^2 in 123 subspace due to the isometry $SO(3)\times SO(6)$, the terms $F_4\wedge F_4$ and $F_4\wedge H_3$ are zero. The equations of motion for the fluxes on the background of the near horizon geometry of a shell of multi-center D0 branes with D2 sources turned on are then modified to:

$$\begin{aligned} dH_3 &= 0, &(110)\\ dG_6 &= J_7, &(111)\\ d[Z_1^{-1}(H_3 - g_s *_9 G_6)] &= 0, &(112)\\ dx^0\wedge d[-g_s^{-1}Z_1^{-1} *_9 H_3 + (Z_1^{-1} - 1)G_6] &= g_s C_1\wedge J_7. &(113) \end{aligned}$$

Since J_7 is a delta function located at the S^2 with radius r_0, so for the right side of equation (113), we need to consider the C_1 at $r_1 = r_0$, $r_2 = 0$. The multi-center warp-factor Z_1 on the location of the delta function is infinity, so $C_1 = -g_s^{-1}dx^0$ on the location of the delta function, where we used the gauge choice $C_1 = g_s^{-1}(Z_1^{-1} - 1)dx^0$. Since $dG_6 = J_7$, the last equation (113) becomes $d[Z_1^{-1}(*_9 H_3 - g_s G_6)] = 0$.

F Solving the equation with source

In this appendix we solve the equations in (41) in section 4. Since $*_9 G_6^{(II)}$ is closed, it can be written as $*_9 G_6^{(II)} = (r_1^{-2}\partial_1 h dr_1 + r_1^{-2}\partial_2 h dr_2) \wedge \omega_2$, so the equation $dG_6^{(II)} = J_7$ gives

$$\partial_1^2 h - 2r_1^{-1}\partial_1 h + \partial_2^2 h + 5r_2^{-1}\partial_2 h = 2\kappa^2 \tau_{D2} g_s^{-2} p r_0^2 \delta(r_1 - r_0)\delta^6(r_2). \tag{114}$$

We can convert equation (114) to a Laplacian equation with source terms by making a derivative with respect to r_1 of both sides of equation (114) and define the function $Y = r_1^{-2}\partial_1 h$. The resulting equation becomes

$$\bigtriangledown_9^2 Y = Q, \tag{115}$$

where $\bigtriangledown_9^2$ is the Laplacian on the 9-d flat space: $\bigtriangledown_9^2 = [r_1^{-2}\partial_1(r_1^2\partial_1) + r_2^{-5}\partial_2(r_2^5\partial_2)]$ and $Q = 2\kappa^2\tau_{D2}g_s^{-2}pr_0^2 r_1^{-2}\delta^{'}(r_1 - r_0)\delta^6(r_2)$ is the source term, where $\delta^{'}(r_1 - r_0)$ is the derivative of $\delta(r_1 - r_0)$ with respect to r_1. Y can be solved by integration via Green's function:

$$Y(\overrightarrow{r}) = \int G(\overrightarrow{r}, \overrightarrow{r}')Q(\overrightarrow{r}')d^9\overrightarrow{r}', \tag{116}$$

where $G(\overrightarrow{r}, \overrightarrow{r}')$ is the green function in the 9-d flat space defined as

$$\begin{aligned} \bigtriangledown_9^2 G(\overrightarrow{r}, \overrightarrow{r}') &= \delta^9(\overrightarrow{r} - \overrightarrow{r}'), & (117)\\ G(\overrightarrow{r}, \overrightarrow{r}') &= \frac{C}{|\overrightarrow{r} - \overrightarrow{r}'|^7}, & (118) \end{aligned}$$

where C is a constant.

Now let's study the Y at a point $(0, 0, r_1, 0, 0, 0, 0, 0, r_2)$. It is the superposition of all the potentials generated by the sources at points parameterized by ($r_1^{'}\sin\theta\cos\varphi$, $r_1^{'}\sin\theta\sin\varphi$, $r_1^{'}\cos\theta$, 0,0,0,0,0,0), where the first three coordinates denote x_1, x_2, x_3 and the last six denote $x_4, ..., x_9$. From equation (116), now Y should be:

$$\begin{aligned} Y &= \int \frac{C}{(r_1^{'2} - 2r_1 r_1^{'}\cos\theta + r_1^2 + r_2^2)^{7/2}} 2\kappa^2\tau_{D2}g_s^{-2}pr_0^2 r_1^{'-2}\delta^{'}(r_1^{'} - r_0)\delta^6(r_2^{'})d^6\overrightarrow{r}^{'}_2 r_1^{'2}dr_1^{'}\sin\theta d\theta d\varphi \\ &= \int \frac{4\pi C\kappa^2\tau_{D2}g_s^{-2}pr_0^2}{5r_1 r_1^{'}}\left(\frac{1}{[(r_1 - r_1^{'})^2 + r_2^2]^{5/2}} - \frac{1}{[(r_1 + r_1^{'})^2 + r_2^2]^{5/2}}\right)\delta^{'}(r_1^{'} - r_0)dr_1^{'}, \quad (119) \end{aligned}$$

where we first integrated over $d^6\overrightarrow{r}'_2$, $d\theta$, $d\varphi$. Then we use $\delta'(r_1' - r_0) = \lim_{\epsilon\to 0}\frac{1}{2\epsilon}[\delta(r_1' - r_0 + \epsilon) - \delta(r_1' - r_0 - \epsilon)]$ and perform the integration and then take $\epsilon \to 0$. We then get the result for Y in equation (43) in section 4.

In the region where $r_1 \gg r_0$, we can expand Y in terms of powers of r_0:

$$Y = \frac{4\pi C\kappa^2\tau_{D2}g_s^{-2}p}{5r_1}\left[\frac{4}{3!}\left(\frac{105r_1}{r^9} - \frac{315r_1^3}{r^{11}}\right)r_0^3 + \frac{8}{5!}\left(\frac{-4527r_1}{r^{11}} + \frac{34650r_1^3}{r^{13}} - \frac{45045r_1^5}{r^{15}}\right)r_0^5 + O(r_0^7)\right]. \tag{120}$$

We see the leading term of (120) is of order r_0^3: $Y = \frac{4\pi C\kappa^2\tau_{D2}g_s^{-2}p}{5r_1}\left[\frac{4}{3!}\left(\frac{105r_1}{r^9} - \frac{315r_1^3}{r^{11}}\right)r_0^3\right] = k\left(\frac{1}{r^9} - \frac{3r_1^2}{r^{11}}\right)r_0^3$, where k is a constant. Then we get $h = k\frac{r_1^3 r_0^3}{3r^9}$ and the leading terms of $H_3^{(II)}$ and $G_6^{(II)}$ are

$$\begin{aligned} H_3^{(II)} &= g_s *_9 G_6^{(1)} = g_s k\frac{r_0^3}{r^9}\left[\left(1 - 3\frac{r_1^2}{r^2}\right)dr_1 \wedge w_2 - 3\frac{r_1 r_2}{r^2}dr_2 \wedge w_2\right] \\ &= g_s k\frac{r_0^3}{r^9}[T_3 - 3V_3], \qquad (121) \\ G_6^{(II)} &= k\frac{r_0^3}{r^9}[*_9 T_3 - *_9 3V_3]. \qquad (122) \end{aligned}$$

We see that the leading terms of $H_3^{(II)}$ and $G_6^{(II)}$ in large r region is precisely our $n = -9$ solution (11) in section 2. Since $r_0 \propto \mu$, they are of the third order in μ in large r region.

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